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A SUGGESTED TECHNIQUE FOR QUANTITATIVE PRECIPITATION FORECASTING

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ABSTRACT

The theory and computation of integrated water vapor transport vectors are described and it is shown how the synoptic analysis of these vectors may be used for 'quantitative precipitation forecasting. An example of the vector field and the precipitation forecast is given. Although the prognostic formula does not give correct point values of the precipitation, reasonably good agreement is found between the distributions of forecast and observed precipitation. The technique is probably too laborious for daily forecasting routine but may be useful in the evaluation of rainmaking experiments.

INTRODUCTION

A variety of techniques for quantitative precipitation forecasting have been developed for use in hydrometeorology (see, for example, Fletcher [1]). The hydrometeorologist is primarily concerned with determining the maximum precipitation that can occur over a given area. The problem of predicting where, when, and how much precipitation will occur falls to the daily short-range weather forecaster who is generally both unfamiliar with and unable to apply the techniques of hydrometeorology. The inability of the daily weather forecaster to apply hydrometeorological techniques to precipitation forecasting is largely due to the fact that these techniques have not been adapted to his requirements.

This problem has been attacked recently by Thompson and Collins [2] who have developed a physical method of computing expected 12-hour rainfall from the fields of wind and humidity. In the method of Thompson and Collins the velocity divergence is computed for 50-mb. layers over a triangular area. The vertical velocity distribution is then computed from the divergence and the precipitation is computed by the method of Fulks [3].

A somewhat different technique is described in the present paper. By integrating in the vertical, water vapor transport vectors are computed from rawinsonde data over a large region of the United States. The continuity condition for water vapor is then employed to calculate the precipitation from the divergence of the integrated water vapor transport. The method has been put to a preliminary test by Locklear [4] who found it to be deficient in several respects. However, since the method is based on what appear to be reasonably sound physical considerations, it is felt that it deserves discussion and further testing. Probably the method will have to be supplemented and modified empirically.

THE CONTINUITY EQUATION FOR WATER VAPOR

Let ρ_v denote the density of water vapor, $\boldsymbol{V}$ the horizontal velocity vector, and w the vertical velocity. In the absence of evaporation and condensation, the continuity equation for water vapor may be written

$$\frac{\partial \rho_v}{\partial t}+\mathrm{div}_2\rho_v\boldsymbol{V}+\frac{\partial \rho_v w}{\partial z}=0 \qquad (1)$$

where z is the vertical coordinate and t denotes time.

Because of the practical difficulties involved in treating evaporation, it will be neglected. But let C denote the mass of water vapor condensed in the air per unit volume. The continuity equation then becomes

$$\frac{\partial \rho_v}{\partial t}+\frac{\partial C}{\partial t}+\mathrm{div}_2\rho_v\boldsymbol{V}+\frac{\partial \rho_v w}{\partial z}=0 \qquad (2)$$

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Next, let us integrate (2) with respect to height between the limits 0 (representing the surface of the earth) and ∞ (representing the top of the atmosphere). In practice, the top of the atmosphere may be taken as some high level above which the water vapor density is inappreciable. Then

$$\frac{\partial}{\partial t}\int_0^\infty \rho_v dz+\frac{\partial}{\partial t}\int_0^\infty C\, dz+\mathrm{div}_2\int_0^\infty \rho_v V\, dz=0 \qquad (3)$$

The vertical divergence term has been eliminated in (3) on the assumption that the vertical velocity vanishes at the surface of the earth and the water vapor density vanishes at "infinity".

The first integral in (3) is known as the precipitable water and is generally denoted by W_p. The second term in (3) represents the condensation rate in the column and will be denoted by I. On the assumption that all condensation products are precipitated, I may be identified with the precipitation intensity. This assumption is not as unreasonable as it may seem in view of the fact that even very deep clouds contain relatively little liquid water compared with the precipitation amounts that occur.

The last integral in (3) will be referred to as the vapor transport vector and will be denoted by $\boldsymbol{F}$. Then, (3) may be written as

$$I=-\frac{\partial W_p}{\partial t}-\mathrm{div}_2 \boldsymbol{F} \qquad (4)$$

The forecaster is generally concerned with predictions of precipitation amounts in finite time intervals (although intensity forecasts may also be demanded). It is therefore necessary to integrate (4) with respect to time. Let the total precipitation in the time interval, $t-t_0$, be denoted by R, where t_0 represents the beginning of the period and t the end. Then,

$$R=-[W_p(t)-W_p(t_0)]-\int_{t_0}^{t} \mathrm{div}_2\, \boldsymbol{F}\, dt \qquad (5)$$

The initial precipitable water, $W_p(t_0)$, can be computed from the radiosonde data. However, the final precipitable water, $W_p(t)$, cannot be predicted and must be estimated by means of some assumption about the condensation-precipitation process. In tests of the method which have been conducted to date (e. g., by Locklear [4]) it has been assumed that $W_p(t)$ is the maximum (saturation) precipitable water corresponding to the initial temperature distribution. This is equivalent to the assumption that no precipitation can fall until the whole column of atmosphere has become saturated Furthermore, the vertical temperature distribution is assumed to be unchanged in the time interval, $t-t_0$. Neither assumption is really very satisfactory. The second assumption can probably be modified by introducing a temperature forecast into the method. But the first assumption leads to an underestimate of the precipitation since the atmosphere is obviously able to produce precipitation from clouds of finite depth. This is one phase of the method which will have to be modified empirically.

The second difficulty in the method is contained in the integral of the divergence of the vapor transport. In the absence of any rational basis for predicting $\mathrm{div}_2\, \boldsymbol{F}$, we are limited to the assumption that this quantity does not change in the interval, $t-t_0$. Whether or not this crude assumption is satisfactory can only be determined by experience.

If we introduce the assumption stated above into (5), we obtain the following formula for the estimation of the precipitation amounts:

$$R=-(W_{ps}-W_p)-(\mathrm{div}_2\, \boldsymbol{F})\ (t-t_0) \qquad (6)$$

where W_{ps} is the saturation value of the precipitable water corresponding to the initial vertical temperature distribution. The quantity $(W_{ps}-W_p)$ will be referred to as the water vapor deficit. Since the water vapor deficit may exceed the convergence of water vapor, R may be negative. A negative value of R is interpreted as zero precipitation for the present. However, when sufficient data have been collected on the method, it may be possible to assign to all values of R (positive and negative) a statement as to the probability of precipitation of any given amount.

PRECIPITABLE WATER

Methods of computing the precipitable water in a column of atmosphere have been described by Solot [5] and others, and the details need not be repeated here.

A simple formula for W_p, expressed in inches of liquid water, may be written to sufficient approximation as

$$W_p=\frac{1}{100}\,[q_1+2(q_2+\ldots+q_{n-1})+q_n] \qquad (7)$$

where q_i, the specific humidity in parts per thousand, is read at 50-mb. intervals beginning at the earth's surface. The corresponding formula for W_{ps} employs the saturation specific humidity.

THE VAPOR TRANSPORT VECTOR

The vapor transport vector, $\boldsymbol{F}$, has been defined above as

$$\boldsymbol{F}=\int_0^\infty \rho_v V\, dz \qquad (8)$$

With the aid of the definition of specific humidity and the hydrostatic equation, (8) may be written

$$\boldsymbol{F}=\int_0^{p_0}\frac{1}{g}\, V q\, dp \qquad (9)$$

where g is the acceleration of gravity and p is the air pressure.

$\boldsymbol{F}$ has the dimensions mass (length) $^{-1}$ (time)$^{-1}$ and the dimensions of $\mathrm{div}_2\, \boldsymbol{F}$ are mass (length)$^{-2}$ (time)$^{-1}$. However, since it is customary to express precipitation amounts in terms of depth rather than mass per unit area, $\boldsymbol{F}$ and $\mathrm{div}_2\, \boldsymbol{F}$ may be divided by the density of liquid water.

The dimensions of these quantities are then (length)^2 (time)$^{-1}$ and length (time)$^{-1}$ respectively.

The calculation of $\boldsymbol{F}$ from rawinsonde data is most conveniently performed by dividing the atmosphere into layers 50-mb. thick, starting at the earth's surface, and integrating numerically. The practical formula for $\boldsymbol{F}$ then becomes

$$\boldsymbol{F}=\frac{50\text{ mb}}{g}\sum_i \overline{\boldsymbol{V}_i q_i} \tag{10}$$

where the bar denotes the mean value of the product in the 50-mb. layer. Upon inserting the values of g and the conversion factor for centimeters to inches, and dividing by the density of liquid water (1 gm. cm.$^{-3}$), we obtain

$$\boldsymbol{F}=\frac{1}{50}\sum_i \overline{\boldsymbol{V}_i q_i} \tag{11}$$

where the units of $\boldsymbol{F}$ and $\boldsymbol{V}_i$ respectively, are (in. mi. hr.$^{-1}$) and (mi. hr.$^{-1}$), and q_i is expressed in parts per thousand. While the use of the mixed units, inches miles (hour)$^{-1}$, appears objectionable, it has the advantage that, when the divergence is computed by finite differences, $\text{div}_2 \boldsymbol{F}$ is easily obtained in the precipitation units, inches (hour)$^{-1}$.

$\boldsymbol{F}$ is computed by tabulating the wind velocities and specific humidities at 50-mb. intervals from the rawinsonde data. The products $\boldsymbol{V}_i q_i$ are also tabulated and the mean values of the products for each 50-mb. layer are obtained. The vectors $\overline{\boldsymbol{V}_i q_i}$ are then added vectorially on a polar diagram to obtain $\boldsymbol{F}$. The decrease of specific humidity with height makes it practicable in most cases to terminate the numerical integration at the 400-mb. level.

The specific humidity distribution can easily be obtained from the dew point curve and the values of $\boldsymbol{V}_i$ can be interpolated with the aid of the pressure-height curve. It would be convenient both for accuracy and efficiency of computation if each rawinsonde station were to compute its own $\boldsymbol{F}$ vector and transmit it with the rawinsonde data. However, this recommendation cannot be considered seriously until the technique has been tested further.

APPLICATION

The calculation of the estimated precipitation, R, from equation (6) can be done on two maps. On the first map is plotted the water vapor deficit, in inches, for each rawinsonde station. Isopleths of $(W_{ps}-W_p)$ are then drawn.

On the second map the values of F_x and F_y, the west-east and south-north components of $\boldsymbol{F}$ in inches miles (hour)$^{-1}$, are plotted for each rawinsonde station. Isopleths of F_x and F_y are drawn using different colors to distinguish the components. A finite difference grid is placed on the map and the divergence of $\boldsymbol{F}$, in inches (hour)$^{-1}$, is computed for each grid point. The values of $\text{div}_2 \boldsymbol{F}$ multiplied by the forecast interval, $t-t_0$, (usually 24 hours) are then plotted at the corresponding grid points on the first map and equation (6) is used to compute R. Isopleths of R may then be constructed on a separate map or overlay to obtain the distribution of the estimated precipitation.

It has been pointed out above that the water vapor deficit factor in (6) may lead to an underestimate of the precipitation. In fact, negative values of R will often be found in regions where measurable amounts of precipitation occur. It is not expected that the values of R will correspond to the precipitation amounts observed in the forecast period. But, if the distribution of R is similar to that of the precipitation, R may be considered a good estimator of the precipitation and empirical relation between R and the precipitation amount may be found.

A sample computation is shown in the figures. The $\boldsymbol{F}$ vectors and the water vapor deficit were computed for 28 radiosonde stations in the eastern half of the United States at 0300 GMT, March 11, 1953. The distribution of $\boldsymbol{F}$ is shown in figure 1. The divergence of $\boldsymbol{F}$ was then calculated by the component method using a grid length of 200 miles. The divergence field is shown in figure 2 where positive values denote divergence of the water vapor flux and negative values denote convergence. Figure 3 shows the distribution of the water vapor deficit.

From figures 2 and 3 the 24-hour rainfall distribution was calculated for the period 0300 GMT, March 11 to 0300 GMT, March 12, 1953. The calculated rainfall distribution is shown in figure 4. The isopleths of expected rainfall have been drawn for a geometric progression of rainfall values beginning with 0.2 inches. The zero line is also shown. But negative values of expected rainfall have been eliminated in the figure. The observed 24-hour rainfall distribution for the period 0630 GMT, March 11, to 0630 GMT, March 12, 1953 (fig. 5) shows that the method did succeed in predicting the major centers of precipitation but failed to give the correct distribution of amounts. Thus, where the method predicted 3.2 inches of rain (in Louisiana and Mississippi) only 0.7 inch was observed, while where the method predicted 0.7 inch (in South

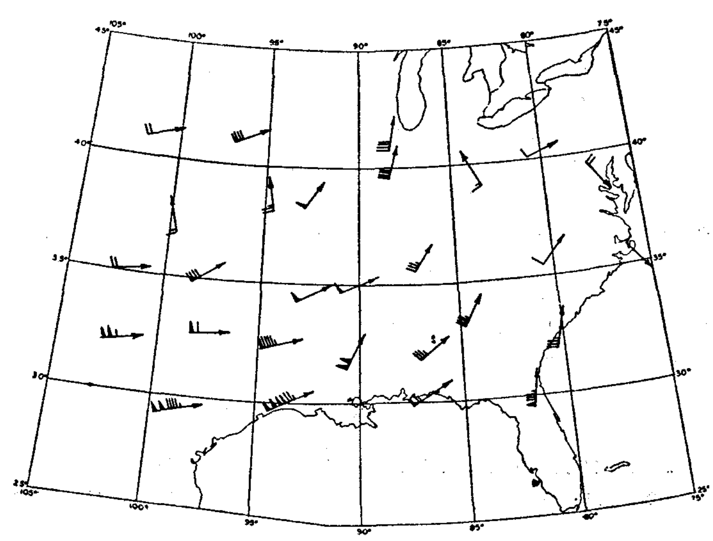

FIGURE 1.—Distribution of water vapor transport vector ($\boldsymbol{F}$), 0300 GMT, March 11, 1953. A full barb represents 2 inch mile (hour)$^{-1}$ and a flag represents 10 in. mi. hr.$^{-1}$.

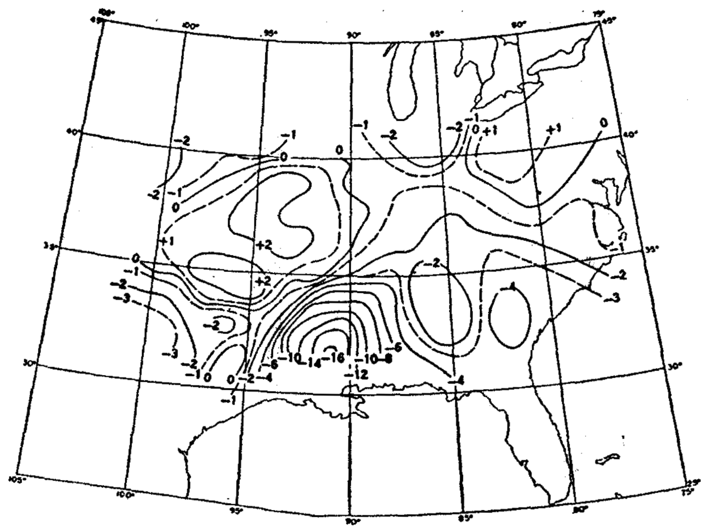

FIGURE 2.—Divergence of water vapor transport (div$_2$ F) 0300 GMT, March 11, 1953 Units are 10^{-2} in. hr.$^{-1}$.

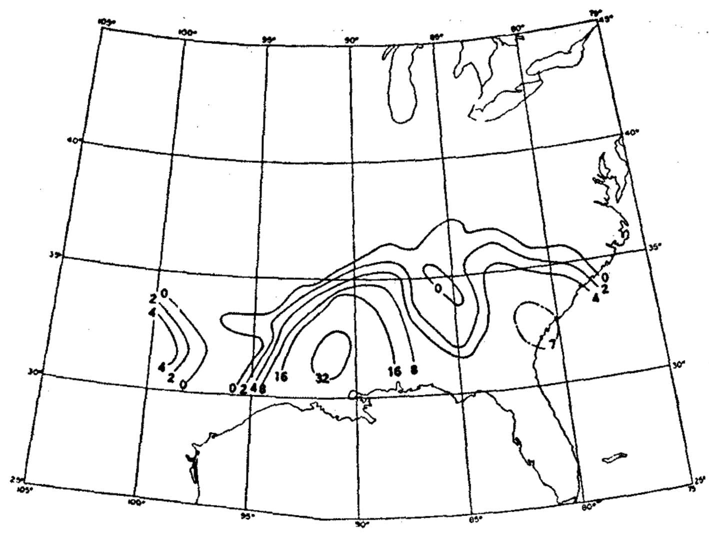

FIGURE 4.—Predicted 24-hour precipitation in tenths of inches, 0300 GMT, March 11, to 0300 GMT March 12, 1953.

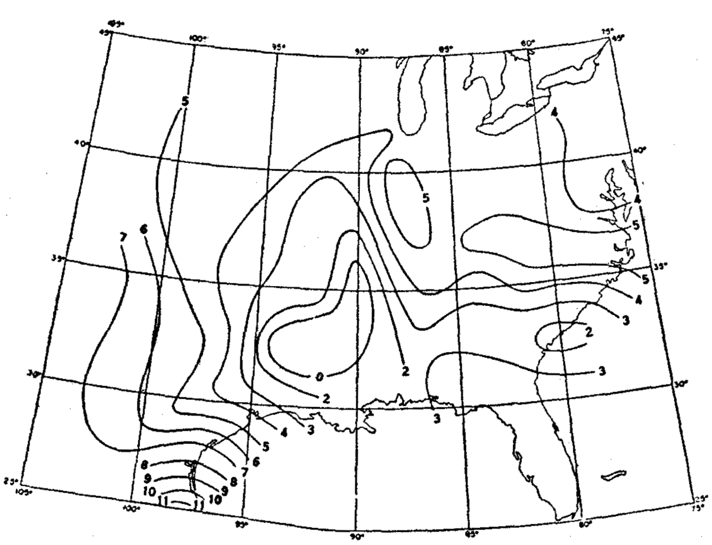

FIGURE 3.—Water vapor deficit ($W_{ps}-W_p$) in tenths of inches.

FIGURE 5.—Observed 24-hour precipitation in tenths of inches, 0630 GMT, March 11, to 0630 GMT, March 12, 1953.

Carolina) more than 1.6 inches were reported. (Savannah recorded 1.95 inches in this period.)

The failures of the method could be due to any of the factors previously mentioned. It appears likely, however, that the major deficiencies of the method are its inability to consider the variations of the divergence of the water vapor transport during the forecast period and the neglect of small scale convergence and convection (i. e., instability). At the present time there appears to be no method of eliminating these difficulties.

RECOMMENDATIONS

The method described above is extremely laborious and time-consuming in its present form and is not practical for use by local forecasting offices. However, it would be possible for a central forecasting office, if sufficient personnel are available, to construct one such prognostic map each day for the entire United States. These prognostic maps could then be distributed by facsimile methods for use by the local forecasting agencies.

The time required for the calculations severely restricts the length of the period for which the forecast is useful. This time could be reduced materially if each rawinsonde station were to compute and transmit with the RAOB report the deficit of precipitable water and the water vapor transport vector. The remaining calculations could then be done in about one hour by an adequate staff.

The method offers some promise as an aid in the evaluation of artificial rain-making experiments. One of the major difficulties of this type of evaluation is the lack of a satisfactory estimate of the natural precipitation to be expected. Although the method presented here may not provide a completely satisfactory forecast, it does give an estimate, based on reasonable physical principles and arrived at objectively, of the expected rainfall. Since the method is not based on average or climatological relationships, it might be useful in the evaluation of rain-making experiments which have been conducted in unusual weather situations, e. g., extremely heavy natural rain storms.

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